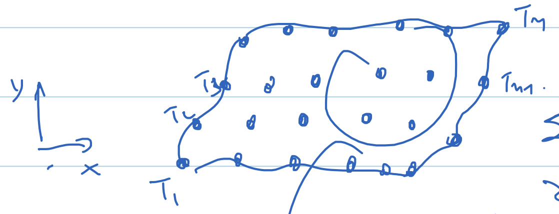
Friday, December 18, 2020
8:30 AM

2D ΠΡΟΒΛΗΜΑΤΑ ΔΙΑΧ. ΘΕΡΜ. ΣΕ ΜΚ.

Προσεγγιστικές Λύσεις

(Πεπερασμένες Διαφορές, Πεπερασμένα Στοιχεία)



Σταθερές ιδιότητες (k).

Σε Μ.Κ. : $k \nabla^2 T = 0 \Rightarrow$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

+ Boundary Conditions } Σχήμα Διακριτοποίησης

↓

Σύστημα αλγεβρικών εξισώσεων.

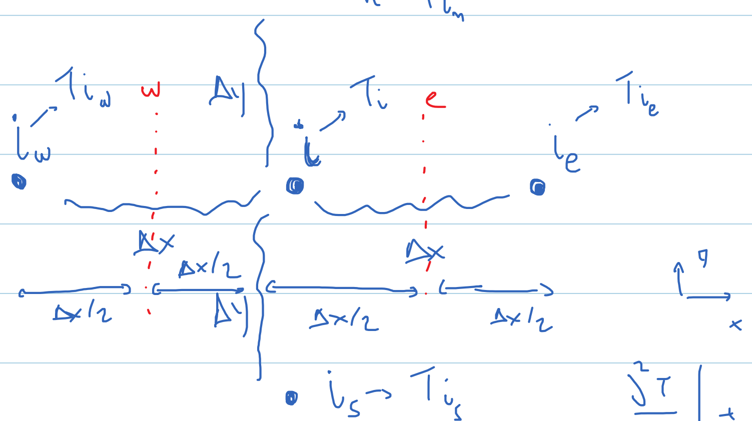
σε κάθε node

$$R_1(T_1, T_2, \dots, T_n) = 0$$

$$R_2(T_1, T_2, \dots, T_n) = 0$$

$$\vdots$$

$$R_n(T_1, T_2, \dots, T_n) = 0$$



$$\frac{\partial^2 T}{\partial x^2}\bigg|_i + \frac{\partial^2 T}{\partial y^2}\bigg|_i = 0.$$

Σχήμα Διακριτοποίησης: Προσεγγίζουμε κεντρικώς.

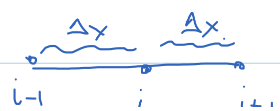
$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \approx \frac{f_e - f_w}{\Delta x}$$

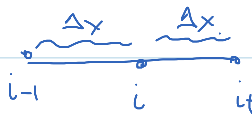
$$\frac{\partial^2 T}{\partial x^2}\bigg|_i \approx \frac{\left(\frac{\partial T}{\partial x}\right)_e - \left(\frac{\partial T}{\partial x}\right)_w}{\Delta x}$$

$$\left(\frac{\partial T}{\partial x}\right)\bigg|_e \approx \frac{T_{i_e} - T_i}{\Delta x}$$

$$\left(\frac{\partial T}{\partial x}\right)_w \approx \frac{T_i - T_{i_w}}{\Delta x}$$

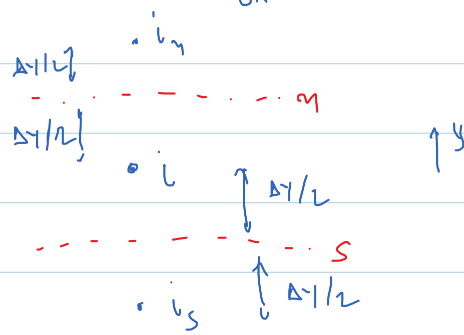
$$\Rightarrow \frac{\partial^2 T}{\partial x^2}\bigg|_i \approx \frac{T_{i_e} + T_{i_w} - 2T_i}{\Delta x^2}.$$





$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1} + T_{i-1} - 2T_i}{\Delta x^2}$$

$$\frac{\partial^2 T}{\partial y^2} \approx \frac{\left(\frac{\partial T}{\partial y}\right)_n - \left(\frac{\partial T}{\partial y}\right)_s}{\Delta y}$$



$$\left(\frac{\partial T}{\partial y}\right)_n \approx \frac{T_{i_n} - T_i}{\Delta y}.$$

$$\left(\frac{\partial T}{\partial y}\right)_s = \frac{T_i - T_{i_s}}{\Delta y}$$

$$\left(\frac{\partial^2 T}{\partial y^2}\right)_i \approx \frac{T_{i_n} + T_{i_s} - 2T_i}{\Delta y^2}.$$

Πεπ. Διαφ.

Άρα: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \Rightarrow \frac{T_{ie} + T_{iw} - 2T_i}{\Delta x^2} + \frac{T_{in} + T_{is} - 2T_i}{\Delta y^2} = 0$

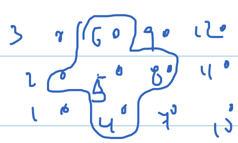
$$\boxed{\frac{1}{\Delta x^2} T_{ie} + \frac{1}{\Delta x^2} T_{iw} + \frac{1}{\Delta y^2} T_{in} + \frac{1}{\Delta y^2} T_{is} - \left(\frac{2}{\Delta x^2} + \frac{2}{\Delta y^2}\right) T_i = 0}$$

Έστω $\Delta x = \Delta y$. $\Rightarrow$ $T_{ie} + T_{iw} + T_{in} + T_{is} - 4T_i = 0$.

Αν χρησιμοποιώ ny κόμβους στην y-διεύθυνση
" " nx -" - " - x - " -

Αριθμηση π.χ. :

$i_n = i+1$
$i_s = i-1$.
$i_e = i + ny$
$i_w = i - ny$.

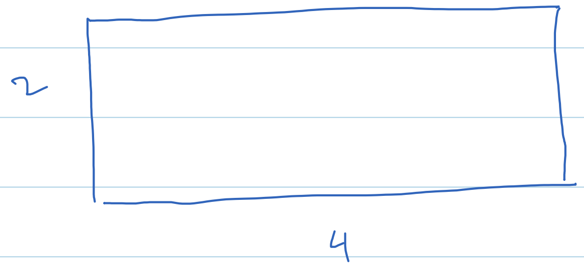


$ny = 3$
$nx = 4$.

Τελικά: $\boxed{T_{i+ny} + T_{i-ny} + T_{i+1} + T_{i-1} - 4T_i = 0}$

2. 6. 5. 8. 4.

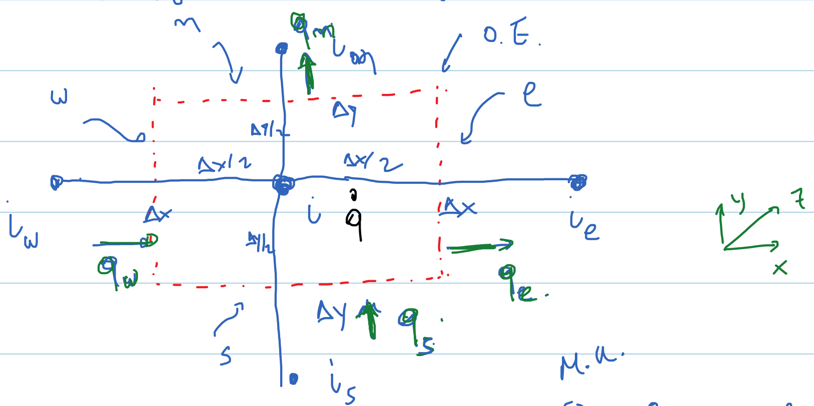
4.



$\Delta x \neq \Delta y$

Energy Balance

Ενεργειακό Ισοζύγιο.



Ενεργειακό Ισοζύγιο του Ο.Ε.: $\overset{\text{Μ.Κ.}\ 0}{\cancel{\dot{E}_{st}}} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$

$$\dot{E}_{in} = q_w + q_s = q''_w \, \Delta y \, (L) + q''_s \cdot \Delta x \, (L)$$

↳ μοναδιαίο βάθος

$$\left.\begin{aligned} q''_w &= -k\left(\frac{\partial T}{\partial x}\right)_w \\ q''_s &= -k\left(\frac{\partial T}{\partial y}\right)_s \end{aligned}\right\} \Rightarrow$$

$$\dot{E}_{in} = -k\left(\frac{\partial T}{\partial x}\right)_w \Delta y (L) - k\left(\frac{\partial T}{\partial y}\right)_s \Delta x (L).$$

$$\left.\begin{aligned} \left(\frac{\partial T}{\partial x}\right)_w &= \frac{T_i - T_w}{\Delta x} \\ \left(\frac{\partial T}{\partial y}\right)_s &= \frac{T_i - T_s}{\Delta y} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \dot{E}_{in} = \frac{k}{\Delta x}(T_w - T_i)\,\Delta y \cdot L + \frac{k}{\Delta y}(T_s - T_i)\,\Delta x \, L.$$

$$\dot{E}_{out} = q_n + q_e = q''_n \Delta x L + q''_e \Delta y L \overset{\text{ν. Fourier}}{=} -k\left(\frac{\partial T}{\partial y}\right)_n \Delta x L - k\left(\frac{\partial T}{\partial x}\right)_e \Delta y L \rightarrow$$

$$\rightarrow \dot{E}_{out} = -k\,\frac{T_n - T_i}{\Delta y}\,\Delta x L - k\,\frac{T_e - T_i}{\Delta x}\,\Delta y L.$$

$$\dot{E}_g = \dot{q}\,\Delta V = \dot{q}\,\Delta x\,\Delta y\,L.$$

$$\frac{k}{}(T_w - T_i)\,\Delta y \cancel{L} + \frac{k}{}(T_s - T_i)\,\Delta x \cancel{L} +$$

$E_{in} - E_{out} + E_g = 0 \Rightarrow$ Δx

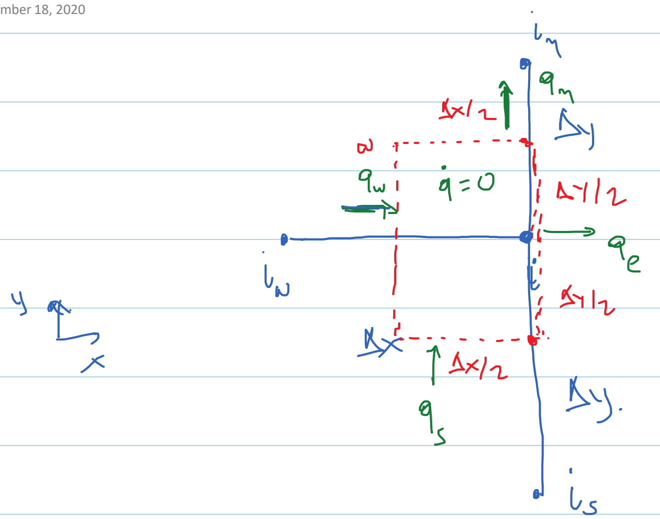
$$\frac{k}{\Delta y}(T_n - T_i)\Delta x \cancel{L} + \frac{k}{\Delta x}(T_e - T_i)\Delta y \cancel{L} + \dot{q}\,\Delta x \Delta y \cancel{L} = 0$$

$$\Rightarrow \frac{k}{\Delta x^2}(T_{i_w} - T_i) + \frac{k}{\Delta y^2}(T_{i_s} - T_i) + \frac{k}{\Delta y^2}(T_{i_n} - T_i) + \frac{k}{\Delta x^2}(T_{i_e} - T_i) + \dot{q} = 0.$$

$$\Downarrow$$

$$\frac{1}{\Delta x^2} T_{i_w} + \frac{1}{\Delta x^2} T_{i_e} + \frac{1}{\Delta y^2} T_{i_n} + \frac{1}{\Delta y^2} T_{i_s} - \left(\frac{2}{\Delta x^2} + \frac{2}{\Delta y^2}\right) T_i + \frac{\dot{q}}{k} = 0.$$

As $\Delta x = \Delta y$. :
$$\boxed{T_{i_w} + T_{i_e} + T_{i_n} + T_{i_s} - 4T_i + \frac{\dot{q}\,\Delta x^2}{k} = 0.}$$



$\uparrow\uparrow\ T_\infty, h$

$$\cancel{\dot E_{st}} = \dot E_{in} - \dot E_{out} + \cancel{\dot E_g}$$

$\dot E_{st}$: 0 (μ.κ.) $\dot E_g$: 0 $(\dot q = 0)$

$$\dot E_{in} - \dot E_{out} = 0.$$

$$\dot E_{in} = q_w + q_s = q''_w \Delta y L + q''_s \frac{\Delta x}{2} L = -k\left(\frac{\partial T}{\partial x}\right)_w \Delta y L - k\left(\frac{\partial T}{\partial y}\right)_s \frac{\Delta x}{2} L.$$

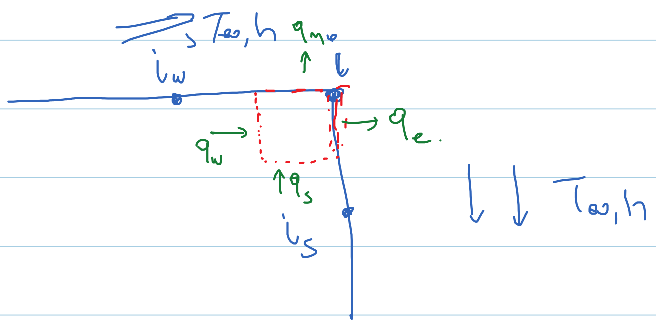
$$\dot E_{out} = q_n + q_e = q''_n \frac{\Delta x}{2} L + q''_e \Delta y L = -k\left(\frac{\partial T}{\partial y}\right)_n \frac{\Delta x}{2} L + h(T_i - T_\infty)\Delta y L$$

$$\dot E_{in} - \dot E_{out} = 0 \Rightarrow -k\left(\frac{\partial T}{\partial x}\right)_w \Delta y \cancel{L} - k\left(\frac{\partial T}{\partial y}\right)_s \frac{\Delta x}{2} \cancel{L} + k\left(\frac{\partial T}{\partial y}\right)_n \frac{\Delta x}{2} \cancel{L} - h(T_i - T_\infty)\Delta y \cancel{L} = 0.$$

$$-k\frac{T_i - T_{iw}}{\Delta x}\Delta y - k\frac{T_i - T_{is}}{\Delta y}\frac{\Delta x}{2} + k\frac{T_{in} - T_i}{\Delta y}\frac{\Delta x}{2} - h(T_i - T_\infty)\Delta y = 0.$$

$$k\frac{\Delta y}{\Delta x} T_{iw} + k\frac{\Delta x}{2\Delta y} T_{is} + \frac{k}{\Delta y}\frac{\Delta x}{2} T_{in} - \left(k\frac{\Delta y}{\Delta x} + k\frac{\Delta x}{\Delta y} + h\Delta y\right) T_i$$
$$+ h T_\infty \Delta y = 0.$$

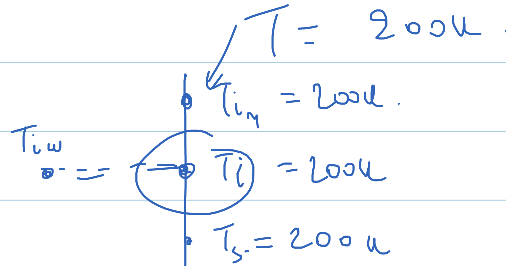
Αν $\Delta y = \Delta x$: $\quad k T_{iw} + \frac{k}{2} T_{is} + \frac{k}{2} T_{in} - (2k + h\Delta y) T_i + h T_\infty \Delta y = 0$



q_n, q_e: Newton

q_w, q_s: Fourier.

Συνοριακή Συνθήκη Dirichlet.



$T_s = 200\,u$