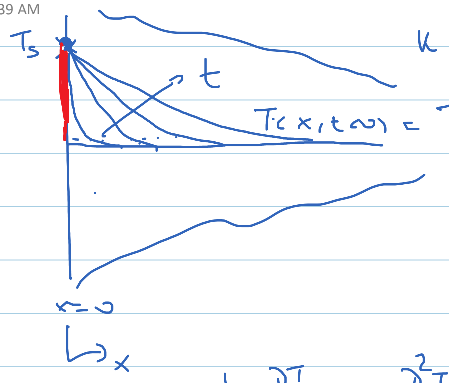
Friday, December 11, 2020
8:39 AM

ΗΜΙΑΠΕΙΡΟ ΣΤΕΡΕΟ.



$k, \rho, c_p.$

$T(x,t)$

$\eta = \dfrac{x}{\sqrt{2\alpha t}} \quad \left(\dot{\eta}\ \dfrac{x}{\sqrt{4\alpha t}} \right)$

$T(\eta)$.

$$\left.\begin{aligned} &\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \\ &\text{Αρχ. Συνθ.}\quad T(x,0) = T_i \\ &\text{Συνορ. Συνθήκη}\ \begin{cases} T(0,t) = T_s \\ T(\infty,t) = T_i \end{cases} \end{aligned}\right\} \Rightarrow \boxed{\frac{T - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right)}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-s^2}\,ds$$

$T = T(x,t)$

Heat flux @ $x=0$: ν. Fourier $\quad q_s'' = -k \cdot \left.\dfrac{\partial T}{\partial x}\right|_{x=0}$ (A).

$$\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left[(T_i - T_s)\operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) + T_s\right] = (T_i - T_s)\frac{\partial}{\partial x}\operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) =$$

$$= (T_i - T_s)\frac{\partial}{\partial x}\left[\frac{2}{\sqrt{\pi}}\int_0^{x/\sqrt{4\alpha t}} \exp(-u^2)\,du\right] =$$

$$= (T_i - T_s)\frac{2}{\sqrt{\pi}}\frac{\partial}{\partial x}\int_0^{x/\sqrt{4\alpha t}} \exp(-u^2)\,du. \quad \left.\right\} =$$

$$\frac{d}{dx}\int_{b(x)}^{c(x)} g(t)\,dt = c'(x)\,g(c(x)) - b'(x)\,g(b(x)).$$

$$= (T_i - T_s)\frac{2}{\sqrt{\pi}} \cdot \frac{d}{dx}\left(\frac{x}{\sqrt{4\alpha t}}\right) \cdot \exp\left(-\left(\frac{x}{\sqrt{4\alpha t}}\right)^2\right) =$$

$$= (T_i - T_s)\frac{1}{\sqrt{\pi\alpha t}}\exp\left(-\left(\frac{x}{\sqrt{4\alpha t}}\right)^2\right).$$

$$q_s'' = -k\left.\frac{\partial T}{\partial x}\right|_{x=0} = -k\frac{T_i - T_s}{\sqrt{\pi\alpha t}} \Rightarrow \boxed{q_s'' = -k\frac{T_i - T_s}{\sqrt{\pi\alpha t}}}$$

ΗΜΙΑΠΕΙΡΟ ΣΤΕΡΕΟ:

Περίπτωση $T(x=0,t) = T_s$: $\dfrac{T - T_s}{\quad} = \operatorname{erf}\left(\dfrac{x}{\quad}\right).$

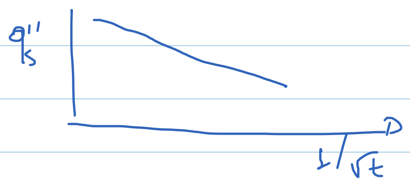
ΗΜΙΑΠΕΙΡΟ ΣΤΕΡΕΟ:

Περίπτωση $T(x=0,t) = T_s$:

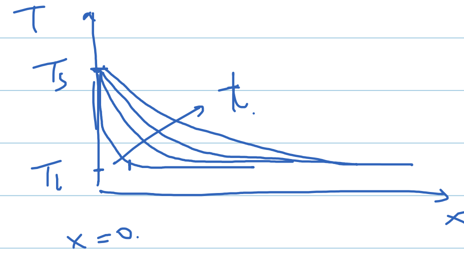
$$\left.\begin{aligned} \frac{T - T_s}{T_i - T_s} &= \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right). \\ q_s'' &= -\frac{k}{\sqrt{\pi\alpha t}} (T_i - T_s) \end{aligned}\right\}$$

$t \to \infty$.

$$\lim_{t\to\infty} \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) = \operatorname{erf}(0) = 0 \Rightarrow T = T_s.$$



ΘΕΡΜΙΚΟ ΠΑΧΟΣ ΔΙΕΙΣΔΥΣΗΣ (THERMAL PENETRATION DEPTH).



Ποιο είναι το βάθος, δ_p, στο οποίο παρατηρούνται σημαντικές θερμοκρασιακές αλλαγές/διαφορές.

$$\frac{T-T_s}{T_i-T_s} \geq 0.9 \Rightarrow T \approx ct = T_i.$$

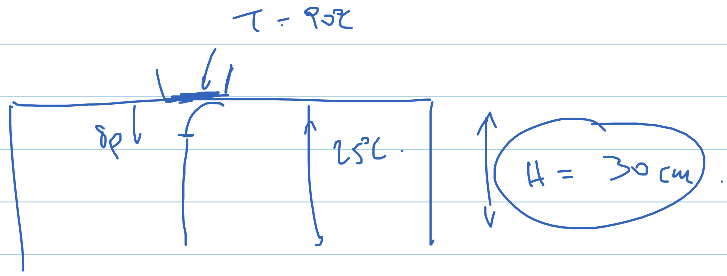
$$\frac{T-T_s}{T_i-T_s} = \operatorname{erf}\left(\frac{\delta_p}{\sqrt{4\alpha t}}\right) = 0.9 \Rightarrow \frac{\delta_p}{\sqrt{4\alpha t}} = 1.163 \Rightarrow$$

$$\Rightarrow \boxed{\delta_p = 2.3\sqrt{\alpha t}}$$

$$\operatorname{erf}(x) = 0.9 \Rightarrow x = 1.163$$

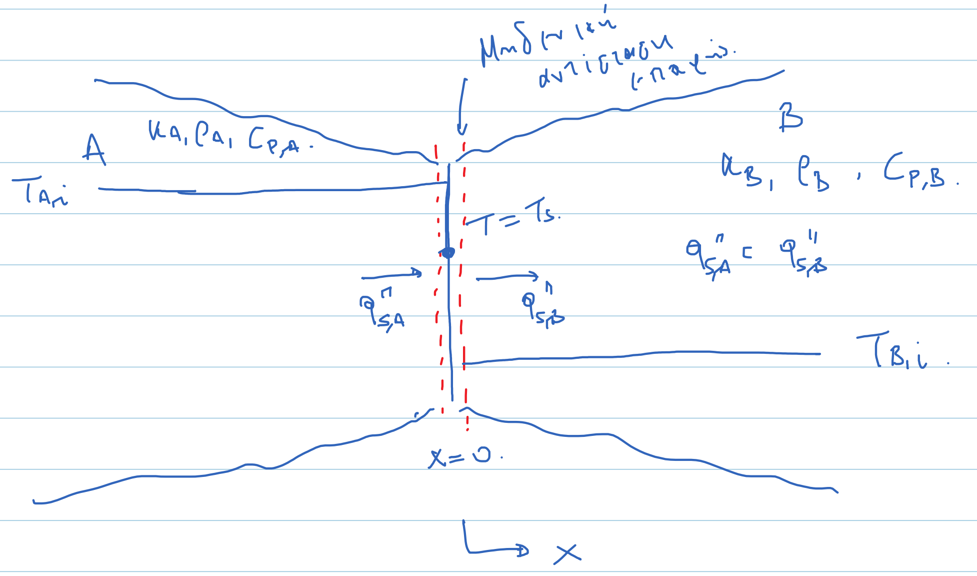
Χρησιμότητα δ_p : ?

Αν $\delta_p \ll H$.
⇓
ισχύει η υπόθεση του ημι-άπειρου στερεού.



Αν L_c : χαρακτηριστική διάσταση ημι-άπειρου στερεού.

Αν $\delta_p(t) \leq L_c \Rightarrow \boxed{2.3\sqrt{\alpha t} \leq L_c.} \Rightarrow$ ισχύει υπόθεση ημι-άπειρου στερεού.



$$q_s'' = -\frac{k}{\sqrt{\pi \alpha t}} (T_i - T_s)$$

$$q'' = -k \frac{\partial T}{\partial x}$$

$$q''_{s,B} = -\frac{k_B}{\sqrt{\pi \alpha_B t}} (T_{B,i} - T_s)$$

$$q''_{s,A} = -\left(-\frac{k_A}{\sqrt{\pi \alpha_A t}} (T_{A,i} - T_s)\right) \Rightarrow$$

$$\Rightarrow \quad + \frac{k_B}{\sqrt{\cancel{\pi} \alpha_B \cancel{t}}} (T_{B,i} - T_s) = +\left(-\frac{k_A}{\sqrt{\cancel{\pi} \alpha_A \cancel{t}}} (T_{A,i} - T_s)\right) \Rightarrow$$

$$\Rightarrow \quad \frac{k_B}{\sqrt{\alpha_B}} (T_{B,i} - T_s) = -\frac{k_A}{\sqrt{\alpha_A}} (T_{A,i} - T_s).$$

$$\alpha = \frac{k}{\rho_B c_p}, \quad \frac{k_B}{\sqrt{\alpha_B}} = \frac{k_B}{\sqrt{\frac{k_B}{\rho_B c_{p,B}}}} = \sqrt{k_B \rho_B c_{p,B}}. \Rightarrow$$

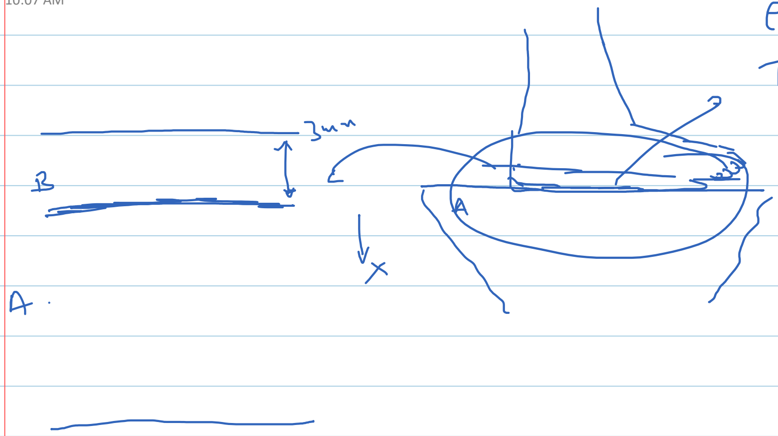
$$\Rightarrow \quad \sqrt{k_B \rho_B c_{p,B}} (T_{B,i} - T_s) = -\sqrt{k_A \rho_A c_{p,A}} (T_{A,i} - T_s). \Rightarrow$$

$$\Rightarrow \quad \boxed{T_s = \frac{\sqrt{k_B \rho_B c_{p,B}}\, T_{B,i} + \sqrt{k_A \rho_A c_{p,A}}\, T_{A,i}}{\sqrt{k_B \rho_B c_{p,B}} + \sqrt{k_A \rho_A c_{p,A}}}}$$

$$T_s = \frac{m_B T_{B,i} + m_A T_{A,i}}{m_B + m_A} \qquad m = \sqrt{k \rho c_p}.$$

Friday, December 11, 2020
10:07 AM

$L_B = 3\,mm.$



$c_{p,B} = 4178\ J/kgK.$

$\rho_B = 993\ kg/m^3.$

$T_{B,i} = 37°C.$

$k_B = 0.3\,W/mK.$

$T_{A,i} = 55°C.$

$\rho_A = 2300\ kg/m^3.$

$k_A = 1.4\ W/mK.$

$c_{p,A} = 880\ J/kgK.$

Παραδοχές: 1D μεταφορά, Αμελητέα Αντίσταση (επαφής), Σταθερές ιδιότητες.

? T στο πέλμα μετά από 1 s.

Ερώτημα: Ισχύει η υπόθεση ημιάπειρου στερεού ?

Πέλμα: $\delta_{p,B} = 2.3\sqrt{\alpha_B t} = \ldots\ldots = 0.625\ mm < 3\,mm.$ Ισχύει.

$$\alpha_B = \frac{k_B}{\rho_B c_{p,B}}, \quad t = 1\,s.$$

Σκυρόδεμα: $\delta_{p,A} = 2.3\sqrt{\alpha_A t} = \ldots = 1.9\times10^{-3}\,m.$ Ισχύει.

$$T_s = \frac{\sqrt{k_A \rho_A c_{p,A}}\cdot \overset{55°C}{T_{A,i}} + \sqrt{k_B \rho_B c_{p,B}}\ \overset{37°C}{T_{B,i}}}{\sqrt{k_A \rho_A c_{p,A}} + \sqrt{k_B \rho_B c_{p,B}}} = 47.8°C.$$

Ελαστικό αδρανές: $\rho_A = 1495\ kg/m^3$, $k_A = 0.28\ W/mK$, $c_{p,A} = 880\ J/kgK.$

$T_s = 43.3°C.$